Open Challenge '24 Solutions

1. Exiled!

The shortest sentence is 1 year which is when the convict is sent to Globe 2 or any odd number Globe greater than 33 (there are 33 of these).
The longest sentence is 27 years which is when the convict is sent to Globe 66.

2. Seeing Stars

A star is splittable if and only if *n* and *m* have a common factor > 1.
In fact if the highest common factor of m and n is h, with *m=ah*, *n=bh*, where *a* and *b* have no common factor > 1, then there are *h* split stars.
The star corresponding to *n* and *m* is identical with the star corresponding to *n-m* and *m*.

n \ m	1	2	3	4	5	6	7	Number of unsplittable
7	✓	✓	✓					3
8	✓	✗	✓	✗				2
9	✓	✓	✗	✓				3
10	✓	✗	✓	✗	✗			2
15	✓	✓	✗	✓	✗	✗	✓	4

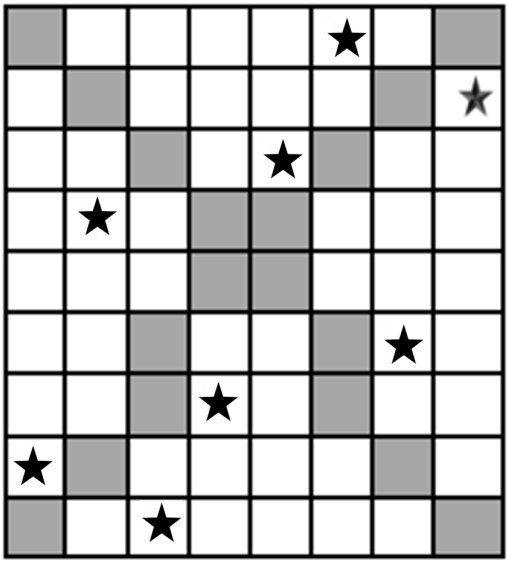
When *n*=42, *m* cannot have a factor of 2, 3 or 7.
Hence there are 6 unsplittable stars with *m*=1, 5, 11, 13, 17 and 19.

3. Star Scatter

This is one of the 18 possible arrangements with the first star fixed.

Without the first star being fixed there are a further 18 arrangements by reflecting horizontally.

Out of the 4 235 364 possible arrangements there are 124 valid ones.



4. From the Earth to the Moon

Please note that the wording in this question implies only a simple solution is necessary and all answers should be given to 2 s.f.

Assume the moon rotates in a circular orbit.

Speed = Distance ÷ Time

3500 ÷ 56 = 62.5 km/min

As 28 days is 40 320 minutes

Distance is 40 320 × 62.5 = 2 520 000 km (circumference of moon's orbit)

Circumference ÷ 2π = 2 500 000 ÷ 2π = 401 070km (radius of orbit)

Radius of orbit – radius of Earth = 401 070 – 6 378

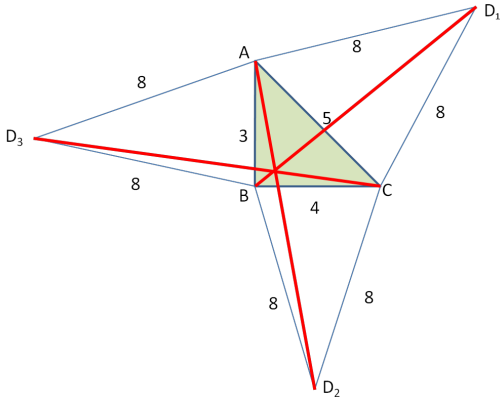
≈ 395 000km

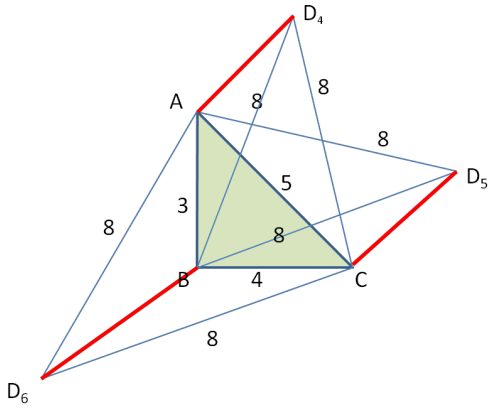
≈ 400 000km (to 2s.f.)

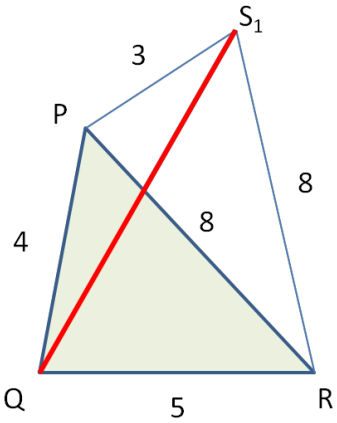
5. How Far to the Star

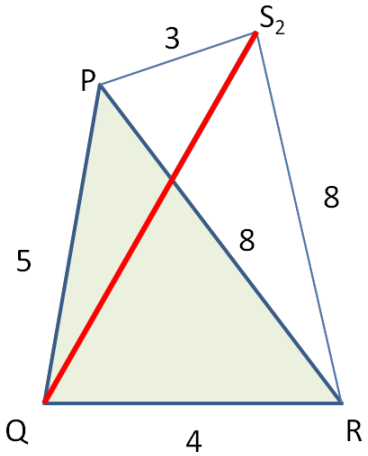
From the given information there are 5 possible triangles that can be formed using three of the stars. 3, 4, 5 3, 8, 8 4, 8, 8 5, 8, 8 and 4, 5, 8

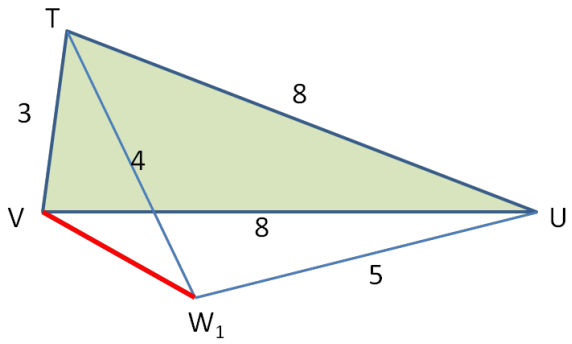
Note that 3, 4, 8 and 3, 5, 8 are not possible.

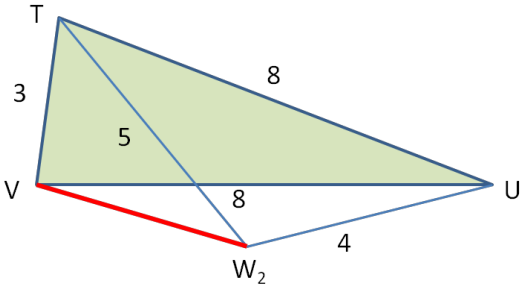


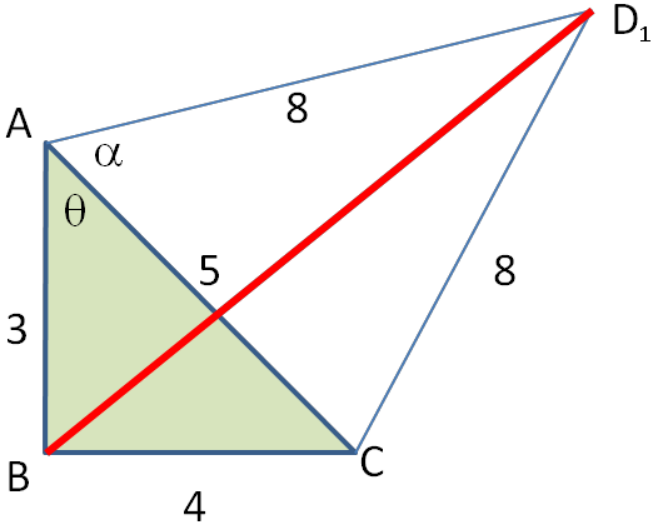












From triangle ABC
$\cos\theta = 3/5$
From triangle ACD_1
$\cos\alpha = 8^2\text{-}8^2\text{-}5^2/(\text{-}2\text{x}5\text{x}8)$
From triangle ABD_1
$(BD_1)^2 = 8^2 + 3^2 - 2\text{x}8\text{x}3\text{x}\cos(\theta+\alpha)$
$(BD_1)^2 = 100.477$

$BD_1 = 10.02$ parsecs (2d.p.)

Similarly all 10 possible values for the sixth distance can be calculated.

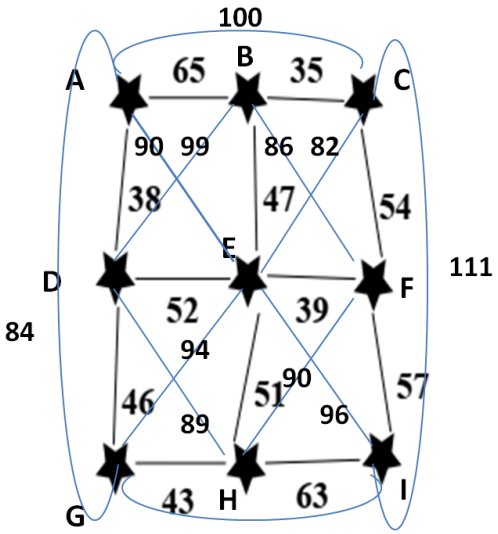
The distances in parsecs to 2d.p. are:

3.01	4.10	4.14	5.15	5.25
5.76	6.40	10.02	10.93	11.95

6. It's in the Stars

Label the stars A to I
A→ H =127, A→ I =186 & A→ F =129
B→ G =141, B→ H =98 & B→ I =143
C→ D =134, C→ G =176 & C→ H =133
D→ F =91, D→ I =148 & G→ F =133

As the furthest distance is 186 A & I are Spica and Kochab (in some order).
Now AE = FH = 90 & CH = FG = 133 and all other distances are unique.
This implies that A must be Spica (and I is Kochab) and E is Unukalhai.
Also F and H are Tejat and Maia (in some order).



As E→ G =94 & E→ B =47 this gives G as Rastaban and B as Procyn
Now as B→ D =99 & B→ H =98 this gives B as Procyon, D as Nunki & H as Maia (hence F is Tejat).
C, the only remaining star, must be Lesath.